51 Cash Flow and Equivalence

Nomenclature

A	annual amount or annual value
C	initial cost, or present worth (present value) of all costs
F	future worth or future value
G	uniform gradient amount
i	interest rate per period
m	number of compounding periods per year
n	number of compounding periods
P	present worth (present value)
r	nominal rate per year (rate per annum)
t	time

Subscripts

0	initial
e	annual effective rate
j	at time j
n	at time n

1. CASH FLOW

The sums of money recorded as receipts or disbursements in a project's financial records are called *cash flows.* Examples of cash flows are deposits to a bank, dividend interest payments, loan payments, operating and maintenance costs, and trade-in salvage on equipment. Whether the cash flow is considered to be a receipt or disbursement depends on the project under consideration. For example, interest paid on a sum in a bank account will be considered a disbursement to the bank and a receipt to the holder of the account.

Because of the time value of money, the timing of cash flows over the life of a project is an important factor. Although they are not always necessary in simple problems (and they are often unwieldy in very complex problems), *cash flow diagrams* can be drawn to help visualize and simplify problems that have diverse receipts and disbursements.

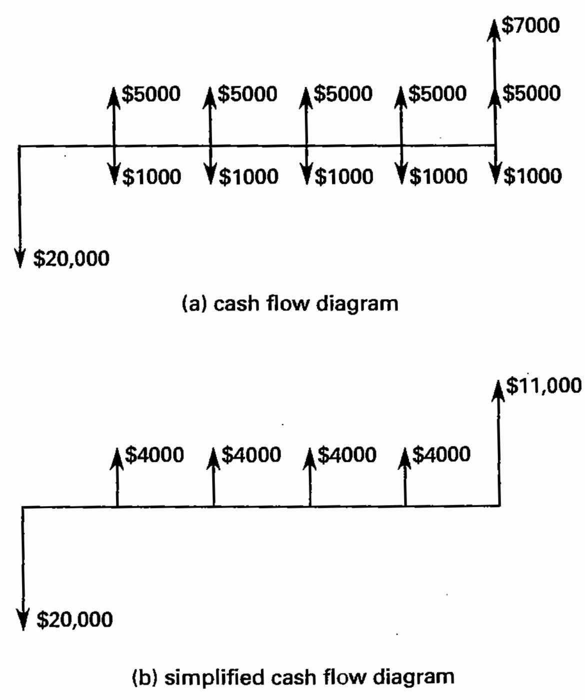
The following conventions are used to standardize cash flow diagrams.

- The horizontal (time) axis is marked off in equal increments, one per period, up to the duration of the project.
- *Receipts* are represented by arrows directed upward. *Disbursements* are represented by arrows directed downward. The arrow length is approximately proportional to the magnitude of the cash flow.
- Two or more transfers in the same period are placed end to end, and these may be combined.
- Expenses incurred before $t=0$ are called *sunk costs.* Sunk costs are not relevant to the problem unless they have tax consequences in an after-tax analysis.

For example, consider a mechanical device that will cost \$20,000 when purchased. Maintenance will cost \$1000 each year. The device will generate revenues of \$5000 each year for five years, after which the salvage value is expected to be \$7000. The cash flow diagram is shown in Fig. 51.1(a), and a simplified version is shown in Fig. 51.1(b).

In order to evaluate a real-world project, it is necessary to present the project's cash flows in terms of standard

Figure 51.1 *Cash Flow Diagrams*



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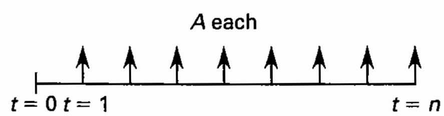
cash flows that can be handled by engineering economic analysis techniques. The standard cash flows are single payment cash flow, uniform series cash flow, and gradient series cash flow.

A *single payment cash flow* can occur at the beginning of the time line (designated as $t=0$), at the end of the time line (designated as $t=n$), or at any time in between.

The *uniform series cash flow*, illustrated in Fig. 51.2, consists of a series of equal transactions starting at $t=1$ and ending at $t=n$. The symbol A (representing an *annual amount*) is typically given to the magnitude of each individual cash flow.

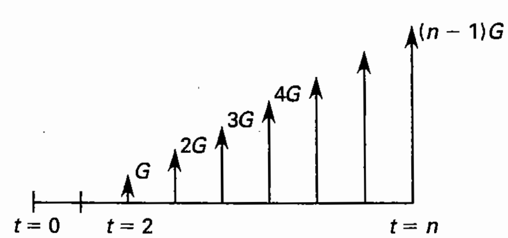
Figure 51.2 *Uniform Series*



Notice that the cash flows do not begin at the beginning of a year (i.e., the year 1 cash flow is at $t=1$, not $t=0$). This convention has been established to accommodate the timing of annual maintenance and other cash flows for which the *year-end convention* is applicable. The year-end convention assumes that all receipts and disbursements take place at the end of the year in which they occur. The exceptions to the year-end convention are *initial project cost* (purchase cost), *trade-in allowance*, and other cash flows that are associated with the inception of the project at $t=0$.

The *gradient series cash flow*, illustrated in Fig. 51.3, starts with a cash flow (typically given the symbol G) at $t=2$ and increases by G each year until $t=n$, at which time the final cash flow is $(n-1)G$. The value of the gradient at $t=1$ is zero.

Figure 51.3 *Gradient Series*



2. TIME VALUE OF MONEY

Consider \$100 placed in a bank account that pays 5% effective annual interest at the end of each year. After the first year, the account will have grown to \$105. After the second year, the account will have grown to \$110.25.

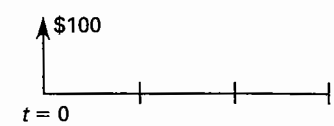
The fact that \$100 today grows to \$105 in one year at 5% annual interest is an example of the *time value of money* principle. This principle states that funds placed in a secure investment will increase in value in a way that depends on the elapsed time and the interest rate.

The interest rate that is used in calculations is known as the *effective interest rate.* If compounding is once a year, it is known as the *effective annual interest rate.* However, effective quarterly, monthly, or daily interest rates are also used.

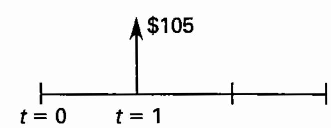
3. DISCOUNT FACTORS AND EQUIVALENCE

Assume that you will have no need for money during the next two years, and any money you receive will immediately go into your account and earn a 5% effective annual interest rate. Which of the following options would be more desirable to you?

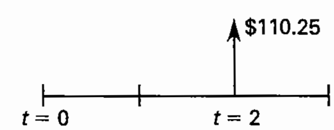
option a: receive \$100 now



option b: receive \$105 in one year



option c: receive \$110.25 in two years



None of the options is superior under the assumptions given. If you choose the first option, you will immediately place \$100 into a 5% account, and in two years the account will have grown to \$110.25. In fact, the account will contain \$110.25 at the end of two years regardless of which option you choose. Therefore, these alternatives are said to be *equivalent.*

The three options are equivalent only for money earning 5% effective annual interest rate. If a higher interest rate can be obtained, then the first option will yield the most money after two years. Thus, equivalence depends on the interest rate, and an alternative that is acceptable to one decision maker may be unacceptable to another who invests at a higher rate. The procedure for determining the equivalent amount is known as *discounting.* For a list of discount factors, see Table 51.1.

Single Payment Equivalence

The equivalent future amount, F, at $t=n$, of any *present amount*, P, at $t=0$ is called the *future worth* and can be calculated from Eq. 51.1. In this equation, and for all the other discounting formulas, the interest rate used must be the effective rate per period. The basis of the

Table 51.1 Discount Factors for Discrete Compounding

factor name	converts	symbol	formula
single payment compound amount	P to F	$(F/P,\ i\%,\ n)$	$(1+i)^n$
single payment present worth	F to P	$(P/F,\ i\%,\ n)$	$(1+i)^{-n}$
uniform series sinking fund	F to A	$(A/F,\ i\%,\ n)$	$\frac{i}{(1+i)^n-1}$
capital recovery	P to A	$(A/P,\ i\%,\ n)$	$\frac{i(1+i)^n}{(1+i)^n-1}$
uniform series compound amount	A to F	$(F/A,\ i\%,\ n)$	$\frac{(1+i)^n-1}{i}$
uniform series present worth	A to P	$(P/A,\ i\%,\ n)$	$\frac{(1+i)^n-1}{i(1+i)^n}$
uniform gradient present worth	G to P	$(P/G,\ i\%,\ n)$	$\frac{(1+i)^n-1}{i^2(1+i)^n}-\frac{n}{i(1+i)^n}$
uniform gradient future worth	G to F	$(F/G,\ i\%,\ n)$	$\frac{(1+i)^n-1}{i^2}-\frac{n}{i}$
uniform gradient uniform series	G to A	$(A/G,\ i\%,\ n)$	$\frac{1}{i}-\frac{n}{(1+i)^n-1}$

rate (annually, monthly, etc.) must agree with the type of period used to count n. It would be incorrect to use an effective annual interest rate if n was the number of compounding periods in months.

$$F = P(1+i)^n \qquad 51.1$$

The factor $(1+i)^n$ is known as the *single payment compound amount factor*.

Similarly, the equivalence of any future amount to any present amount is called the *present worth* and can be calculated from Eq. 51.2.

$$P = F(1+i)^{-n} = \frac{F}{(1+i)^n} \qquad 51.2$$

The factor $(1+i)^{-n}$ is known as the *single payment present worth factor*.

Rather than actually writing the formula for the compound amount factor (which converts a present amount to a future amount), it is common convention to substitute the standard functional notation of $(F/P,\ i\%,\ n)$. This notation is interpreted as, "Find F, given P, using an interest rate of $i\%$ over n years." Thus, the future value in n periods of a present amount would be symbolically written as

$$F = P(F/P,\ i\%,\ n) \qquad 51.3$$

Similarly, the present worth factor has a functional notation of $(P/F,\ i\%,\ n)$. The present worth of a future amount n periods from now would be symbolically written as

$$P = F(P/F,\ i\%,\ n) \qquad 51.4$$

The discounting factors are listed in Table 51.1 in symbolic and formula form. Normally, it will not be necessary to calculate factors from these formulas. Values of these cash flow (discounting) factors are tabulated in Table 51.2 through Table 51.11 for various combinations of i and n. For intermediate values, computing the factors from the formulas may be necessary, or linear interpolation can be used as an approximation.

Uniform Series Equivalence

A cash flow that repeats at the end of each year for n years without change in amount is known as an *annual amount* and is given the symbol A. (This is shown in Fig. 51.2.) Although the equivalent value for each of the n annual amounts could be calculated and then summed, it is more expedient to use one of the uniform series factors. For example, it is possible to convert from an annual amount to a future amount by using the *(F/A) uniform series compound amount factor*.

$$F = A(F/A,\ i\%,\ n) \qquad 51.5$$

Example 51.1

Suppose you deposited $200 at the end of every year for seven years in an account that earned 6% annual effective interest. At the end of seven years, how much would the account be worth?

Solution

Using Eq. 51.5,

$$\begin{aligned} F &= (\$200)(F/A, 6\%, 7) \\ &= (\$200)\left(\frac{(1+0.06)^7 - 1}{0.06}\right) \\ &= (\$200)(8.3938) \\ &= \$1678.76 \end{aligned}$$

(The value of 8.3938 could easily have been obtained directly from Table 51.7 at the end of this chapter.)

A *sinking fund* is a fund or account into which annual deposits of A are made in order to accumulate F at $t=n$ in the future. Because the annual deposit is calculated as $A=F(A/F, i\%, n)$, the (A/F) factor is known as the *sinking fund factor.*

Example 51.2

Suppose you want exactly $1600 in the previous investment account at the end of the seventh year. By using the sinking fund factor, you could calculate the necessary annual amount you would need to deposit.

Solution

From Table 51.1,

$$\begin{aligned} A &= F(A/F, 6\%, 7) \\ &= (\$1600)\left(\frac{0.06}{(1+0.06)^7 - 1}\right) \\ &= (\$1600)(0.1191) \\ &= \$190.56 \end{aligned}$$

An *annuity* is a series of equal payments, A, made over a period of time. Usually, it is necessary to "buy into" an investment (a bond, an insurance policy, etc.) in order to fund the annuity. In the case of an annuity that starts at the end of the first year and continues for n years, the purchase price, P, would be

$$P = A(P/A, i\%, n) \qquad 51.6$$

Example 51.3

Suppose you will retire in exactly one year and want an account that will pay you $20,000 a year for the next 15 years. (The fund will be depleted at the end of the fifteenth year.) Assuming a 6% annual effective interest rate, what is the amount you would need to deposit now?

Solution

Using Eq. 51.6,

$$\begin{aligned} P &= A(P/A, 6\%, 15) \\ &= (\$20{,}000)\left(\frac{(1+0.06)^{15} - 1}{(0.06)(1+0.06)^{15}}\right) \\ &= (\$20{,}000)(9.7122) \\ &= \$194{,}244 \end{aligned}$$

Gradient Equivalence

If the cash flow has the proper form (i.e., Fig. 51.3), its present worth can be determined by using the *uniform gradient factor*, $(P/G, i\%, n)$. The uniform gradient factor finds the present worth of a uniformly increasing cash flow. By definition of a uniform gradient, the cash flow starts in year 2, not in year 1.

There are three common difficulties associated with the form of the uniform gradient. The first difficulty is that the first cash flow starts at $t=2$. This convention recognizes that annual costs, if they increase uniformly, begin with some value at $t=1$ (due to the year-end convention), but do not begin to increase until $t=2$. The tabulated values of (P/G) have been calculated to find the present worth of only the increasing part of the annual expense. The present worth of the base expense incurred at $t=1$ must be found separately with the (P/A) factor.

The second difficulty is that, even though the $(P/G, i\%, n)$ factor is used, there are only $n-1$ actual cash flows. n must be interpreted as the *period number* in which the last gradient cash flow occurs, not the number of gradient cash flows.

Finally, the sign convention used with gradient cash flows may seem confusing. If an expense increases each year, the gradient will be negative, since it is an expense. If a revenue increases each year, the gradient will be positive. In most cases, the sign of the gradient depends on whether the cash flow is an expense or a revenue.

Example 51.4

A bonus package pays an employee $1000 at the end of the first year, $1500 at the end of the second year, and so on, for the first nine years of employment. What is the present worth of the bonus package at 6% interest?

Solution

The present worth of the bonus package is

$$\begin{aligned} P &= (\$1000)(P/A,\ 6\%,\ 9) + (\$500)(P/G,\ 6\%,\ 9) \\ &= (\$1000)\left(\frac{(1+0.06)^9 - 1}{(0.06)(1+0.06)^9}\right) \\ &\quad + (\$500)\left(\frac{(1+0.06)^9 - 1}{(0.06)^2(1+0.06)^9} - \frac{9}{(0.06)(1+0.06)^9}\right) \\ &= (\$1000)(6.8017) + (\$500)(24.5768) \\ &= \$19{,}090 \end{aligned}$$

4. FUNCTIONAL NOTATION

There are several ways of remembering what the functional notation means. One method of remembering which factor should be used is to think of the factors as *conditional probabilities*. The conditional probability of event A given that event B has occurred is written as $P\{A|B\}$, where the given event comes after the vertical bar. In the standard notational form of discounting factors, the given amount is similarly placed after the slash. What you want, A, comes before the slash. (F/P) would be a factor to find F given P.

Another method of remembering the notation is to interpret the factors algebraically. The (F/P) factor could be thought of as the fraction F/P. The numerical values of the discounting factors are consistent with this algebraic manipulation. The (F/A) factor could be calculated as $(F/P)(P/A)$. This consistent relationship can be used to calculate other factors that might be occasionally needed, such as (F/G) or (G/P). For instance, the annual cash flow that would be equivalent to a uniform gradient may be found from

$$A = G(P/G,\ i\%,\ n)(A/P,\ i\%,\ n) \qquad 51.7$$

5. NONANNUAL COMPOUNDING

If \$100 is invested at 5%, it will grow to \$105 in one year. If only the original principal accrues interest, the interest is known as *simple interest* and the account will grow to \$110 in the second year, \$115 in the third year, and so on. Simple interest is rarely encountered in engineering economic analyses.

More often, both the principal and the interest earned accrue interest, and this is known as *compound interest*. If the account is compounded yearly, then during the second year, 5% interest continues to be accrued, but on \$105, not \$100, so the value at year end will be \$110.25. The value after the third year will be \$115.76, and so on.

The interest rate used in the discount factor formulas is the *interest rate per period*, i (called the *yield* by banks). If the interest period is one year (i.e., the interest is compounded yearly), then the interest rate per period, i, is equal to the *annual effective interest rate*, i_e. The annual effective interest rate is the rate that would yield the same accrued interest at the end of the year if the account were compounded yearly.

The term *nominal interest rate*, r (*rate per annum*), is encountered when compounding is more than once per year. The nominal rate does not include the effect of compounding and is not the same as the annual effective interest rate.

The effective interest rate can be calculated from the nominal rate if the number of compounding periods per year is known. If there are m compounding periods during the year (two for semiannual compounding, four for quarterly compounding, twelve for monthly compounding, etc.), the *effective interest rate per period*, i, is r/m. The effective annual interest rate, i_e, can be calculated from the interest rate per period by using Eq. 51.9.

$$i = \frac{r}{m} \qquad 51.8$$

$$\begin{aligned} i_e &= (1+i)^m - 1 \\ &= \left(1 + \frac{r}{m}\right)^m - 1 \end{aligned} \qquad 51.9$$

Sometimes, only the effective rate per period (e.g., per month) is known. However, compounding for m periods at an effective interest rate per period is not affected by the definition or length of the period. For example, compounding for 365 periods (days) at an interest rate of 0.03808% is the same as compounding for 12 periods (months) at an interest rate of 1.164%, or once at an effective annual interest rate of 14.9%. In each case, the interest rate per period is different, but the effective annual interest rate is the same. If only the daily effective rate were given, the discount factor formulas could be used with $i = 0.03808\%$ and $n = 365$ to represent each yearly cash flow. Equation 51.9 could be used to calculate $i_e = 14.9\%$ to use with $n = 1$ for each yearly cash flow.

Since they do not account for the effect of compounding, nominal rates cannot be compared unless the method of compounding is specified. The only practical use for a nominal rate is for calculating the effective rate.

The following rules may be used to determine what type of interest rate is given in a problem.

- Unless specifically qualified in the problem, the interest rate given is an annual rate. If the compounding period is not specified, the interest rate is the annual effective interest rate, i_e.

- If the compounding is annual, the rate given is the effective rate, i_e. If compounding is not annual, the rate given is the nominal rate, r.

6. CONTINUOUS COMPOUNDING

Discount factors for continuous compounding are different from those for discrete compounding. The discounting factors can be calculated directly from the nominal interest rate, r, and number of years, n, without having to find the effective interest rate per period.

$$(F/P,\ r\%,\ n) = e^{rn} \qquad 51.10$$

$$(P/F,\ r\%,\ n) = e^{-rn} \qquad 51.11$$

$$(A/F,\ r\%,\ n) = \frac{e^r - 1}{e^{rn} - 1} \qquad 51.12$$

$$(F/A,\ r\%,\ n) = \frac{e^{rn} - 1}{e^r - 1} \qquad 51.13$$

$$(A/P,\ r\%,\ n) = \frac{e^r - 1}{1 - e^{-rn}} \qquad 51.14$$

$$(P/A,\ r\%,\ n) = \frac{1 - e^{-rn}}{e^r - 1} \qquad 51.15$$

The effective annual interest rate determined on a daily compounding basis will not be significantly different than if continuous compounding is assumed.

SAMPLE PROBLEMS

Problem 1

If a credit union pays 4.125% interest compounded quarterly, what is the effective annual interest rate?

(A) 4.189%

(B) 8.250%

(C) 12.89%

(D) 17.55%

Solution

4.125% is the nominal annual rate, r.

$$\begin{aligned} i^e &= (1 + i)^m - 1 \\ &= \left(1 + \frac{0.04125}{4}\right)^4 - 1 \\ &= 0.04189 \quad (4.189\%) \end{aligned}$$

Answer is A.

Problem 2

The national debt is approximately \$4 trillion. What is the required payment per year to completely pay off the debt in 20 years, assuming an interest rate of 6%?

(A) \$315 billion

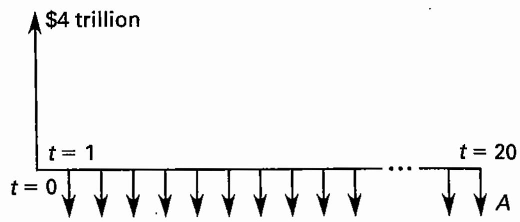
(B) \$325 billion

(C) \$350 billion

(D) \$415 billion

Solution

Create a cash flow diagram.



Use the capital recovery discount factor from the tables.

$$(A/P,\ 6\%,\ 20) = 0.0872$$

$$\begin{aligned} A &= P(A/P,\ 6\%,\ 20) \\ &= (\$4{,}000{,}000{,}000{,}000)(0.0872) \\ &= \$348{,}800{,}000{,}000 \quad (\$350 \text{ billion}) \end{aligned}$$

Answer is C.

Problem 3

The president of a growing engineering firm wishes to give each of 50 employees a holiday bonus. How much is needed to invest monthly for a year at 12% nominal interest rate, compounded monthly, so that each employee will receive a \$1000 bonus?

(A) \$2070

(B) \$3840

(C) \$3940

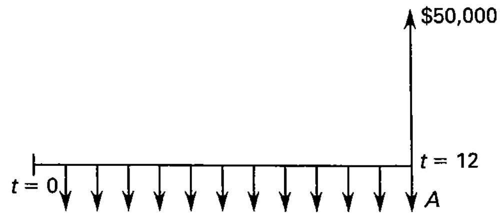
(D) \$4170

Solution

The total holiday bonus is

$$(50)(\$1000) = \$50{,}000$$

Economics



Use the uniform series sinking fund discount factor. The interest period is one month, there are 12 compounding periods, and the effective interest rate per interest period is 12%/12 = 1%.

$$(A/F,\ 1\%,\ 12) = \frac{0.01}{(1+0.01)^{12}-1} = 0.0788$$

$$A = F(A/F,\ 1\%,\ 12)$$
$$= (\$50{,}000)(0.0788)$$
$$= \$3940$$

Answer is C.

Problem 4

If the nominal interest rate is 3%, how much is $5000 worth in 10 years in a continuously compounded account?

(A) $3180

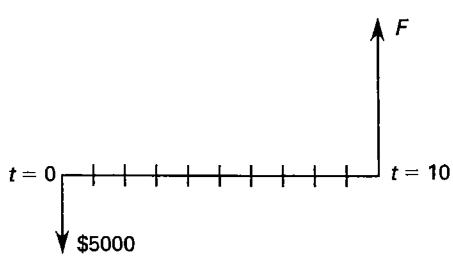
(B) $4490

(C) $5420

(D) $6750

Solution

Create a cash flow diagram.



Use the single payment compound amount factor for continuous compounding.

$$n = 10$$
$$r = 3\%$$
$$(F/P,\ r\%,\ n) = e^{rn}$$
$$(F/P,\ 3\%,\ 10) = e^{(0.03)(10)} = 1.34986$$
$$F = P(F/P,\ 3\%,\ 10)$$
$$= (\$5000)(1.34986)$$
$$= \$6749$$

Answer is D.

Problem 5

An engineering graduate plans to buy a home. She has been advised that her monthly house and property tax payment should not exceed 35% of her disposable monthly income. After researching the market, she determines she can obtain a 30-year home loan for 6.95% annual interest per year, compounded monthly. Her monthly property tax payment will be approximately $150. What is the maximum amount she can pay for a house if her disposable monthly income is $2000?

(A) $80,000

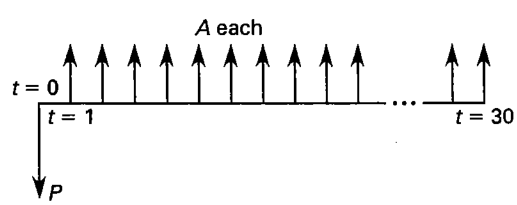
(B) $83,100

(C) $85,200

(D) $90,500

Solution

Create a cash flow diagram.



The amount available for monthly house payments, A, is

$$(\$2000)(0.35) - \$150 = \$550$$

Use the uniform series present worth discount factor. The effective rate per period is

$$i = \frac{0.0695}{12 \text{ months}} = 0.00579 \text{ per month}$$

$$n = (30 \text{ years})\left(12\ \frac{\text{months}}{\text{year}}\right) = 360 \text{ months}$$

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There are no tables for this interest rate.

$$(P/A, 0.579\%, 360) = \frac{(1+i)^n - 1}{i(1+i)^n}$$
$$= \frac{(1+0.00579)^{360} - 1}{(0.00579)(1+0.00579)^{360}}$$
$$= 151.10$$
$$P = A(P/A, 0.579\%, 360)$$
$$= (\$550)(151.10)$$
$$= \$83,105$$

Answer is B.

Problem 6

The designer of the penstock for a small hydroelectric cogeneration station has the option of using steel pipe, which costs $150,000 installed and requires $5000 yearly for painting and leak-checking maintenance, or DSR4.3 (heavy-duty plastic) pipe, which costs $180,000 installed and requires $1200 yearly for leak-checking maintenance. Both options have an expected life of 25 years. If the interest rate is 8%, which choice has the lower present equivalent cost and how much lower is it?

(A) DSR4.3 costs less by $10,600.

(B) Steel pipe costs less by $10,600.

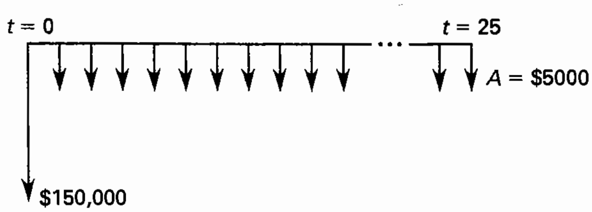
(C) DSR4.3 costs less by $65,000.

(D) Steel pipe costs less by $65,000.

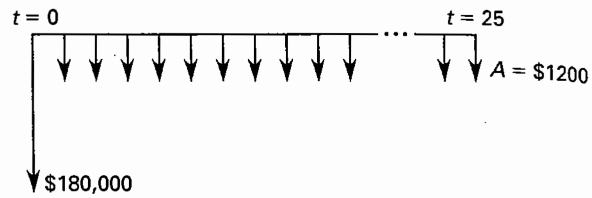
Solution

The problem requires a comparison of the uniform series present worth of each alternative.

For steel pipe,



For DSR4.3,



$$P(\text{steel pipe}) = \$150,000 + A(P/A, 8\%, 25)$$
$$= \$150,000 + (\$5000)(10.6748)$$
$$= \$203,374$$
$$P(\text{DSR4.3}) = \$180,000 + A(P/A, 8\%, 25)$$
$$= \$180,000 + (\$1200)(10.6748)$$
$$= \$192,810$$

Using DSR4.3 is less expensive by

$$\$203,374 - \$192,810 = \$10,564 \quad (\$10,600)$$

Answer is A.

FE-STYLE EXAM PROBLEMS

1. If the interest rate on an account is 11.5% compounded yearly, approximately how many years will it take to triple the amount?

(A) 8 years

(B) 9 years

(C) 10 years

(D) 11 years

2. Fifteen years ago $1000 was deposited in a bank account, and today it is worth $2370. The bank pays interest semi-annually. What was the nominal annual interest rate paid on this account?

(A) 2.9%

(B) 4.4%

(C) 5.0%

(D) 5.8%

3. Mr. Jones plans to deposit $500 at the end of each month for 10 years at 12% annual interest, compounded monthly. The amount that will be available in two years is

(A) $13,000

(B) $13,500

(C) $14,000

(D) $14,500

4. The purchase price of a car is \$25,000. Mr. Smith makes a down payment of \$5000 and borrows the balance from a bank at 6% annual interest, compounded monthly for five years. Calculate the nearest value of the required monthly payments to pay off the loan.

(A) \$350

(B) \$400

(C) \$450

(D) \$500

5. A piece of machinery can be bought for \$10,000 cash or for \$2000 down and payments of \$750 per year for 15 years. What is the annual interest rate for the time payments?

(A) 1.51%

(B) 4.61%

(C) 7.71%

(D) 12.0%

For the following problems, use the NCEES Handbook as your only reference.

6. You have borrowed \$5000 and must pay it off in five equal annual payments. Your annual interest rate is 10%. How much interest will you pay in the first two years?

(A) \$855

(B) \$868

(C) \$875

(D) \$918

7. A company puts \$25,000 down and will pay \$5000 every year for the life of a machine (10 years). If the salvage value is zero and the interest rate is 10% compounded annually, what is the present value of the machine?

(A) \$55,700

(B) \$61,400

(C) \$75,500

(D) \$82,500

8. You borrow \$3500 for one year from a friend at an interest rate of 1.5% per month instead of taking a loan from a bank at a rate of 18% per year. Compare how much money you will save or lose on the transaction.

(A) You will pay \$55 more than if you borrowed from the bank.

(B) You will pay \$630 more than if you borrowed from the bank.

(C) You will pay \$685 more than if you borrowed from the bank.

(D) You will save \$55 by borrowing from your friend.

9. If you invest \$25,000 at 8% interest compounded annually, approximately how much money will be in the account at the end of 10 years?

(A) \$31,000

(B) \$46,000

(C) \$54,000

(D) \$75,000

10. A college student borrows \$10,000 today at 10% interest compounded annually. Four years later, the student makes the first repayment of \$3000. Approximately how much money will the student still owe on the loan after the first payment?

(A) \$7700

(B) \$8300

(C) \$11,000

(D) \$11,700

11. A 40-year-old consulting engineer wants to set up a retirement fund to be used starting at age 65. \$20,000 is invested now at 6% compounded annually. Approximately how much money will be in the fund at retirement?

(A) \$84,000

(B) \$86,000

(C) \$88,000

(D) \$92,000

12. The maintenance cost for a car this year is expected to be $500. The cost will increase $50 each year for the subsequent 9 years. The interest is 8% compounded annually. What is the approximate present worth of maintenance for the car over the full 10 years?

(A) $4300

(B) $4700

(C) $5300

(D) $5500

13. A house is expected to have a maintenance cost of $1000 the first year. It is believed that the maintenance cost will increase $500 per year. The interest rate is 6% compounded annually. Over a 10-year period, what will be the approximate effective annual maintenance cost?

(A) $1900

(B) $3000

(C) $3500

(D) $3800

14. You deposited $10,000 in a savings account five years ago. The account has earned 5.25% interest compounded continuously since then. How much money is in the account today?

(A) $12,800

(B) $12,900

(C) $13,000

(D) $13,600

15. A young engineer wants to surprise her husband with a European vacation for their tenth anniversary, which is five years away. She determines that the trip will cost $5000. Assuming an interest rate of 5.50% compounded daily, approximately how much money does she need to deposit today for the trip?

(A) $3790

(B) $3800

(C) $3880

(D) $3930

16. A young woman plans to retire in 30 years. She intends to contribute the same amount of money each year to her retirement fund. The fund earns 10% compounded annually. She would like to withdraw $100,000 each year for 20 years, starting 1 year after the last contribution is made. Approximately how much money should she contribute to her retirement fund each year?

(A) $490

(B) $570

(C) $5200

(D) $11,000

17. A deposit of $1000 is made in a bank account that pays 8% interest compounded annually. Approximately how much money will be in the account after 10 years?

(A) $1890

(B) $2000

(C) $2160

(D) $2240

18. A deposit of $1000 is made in a bank account that pays 24% interest per year compounded quarterly. Approximately how much money will be in the account after 10 years?

(A) $7000

(B) $7200

(C) $8600

(D) $10,000

19. A machine costs $20,000 today and has an estimated scrap cash value of $2000 after eight years. Inflation is 8% per year. The effective annual interest rate earned on money invested is 6%. How much money needs to be set aside each year to replace the machine with an identical model eight years from now?

(A) $2970

(B) $3000

(C) $3290

(D) $3540

20. At what rate of annual interest will an investment quadruple itself in 12 years?

(A) 10.1%

(B) 11.2%

(C) 12.2%

(D) 13.1%

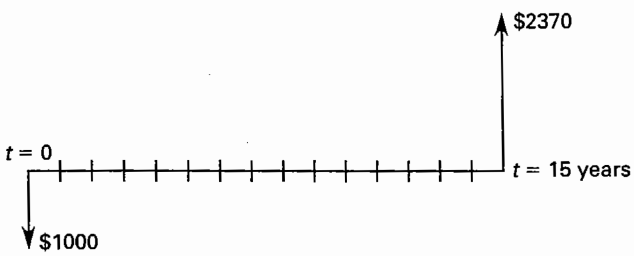
SOLUTIONS

1. The future amount will be three times the present amount when the (F/P) factor is equal to 3.

$$\begin{aligned}(F/P,\ i\%,\ n) &= (1+i)^n\\ (1+0.115)^n &= 3\\ n\log 1.115 &= \log 3\\ n &= \frac{\log 3}{\log 1.115}\\ &= 10.09\ \text{years} \quad (10\ \text{years})\end{aligned}$$

Answer is C.

2. Create a cash flow diagram.



$$\begin{aligned}P &= \$1000\\ n &= (15\ \text{years})(2\ \text{compounding periods per year})\\ &= 30\ \text{compounding periods}\\ F &= P(F/P,\ i\%,\ n)\\ \$2370 &= (\$1000)(F/P,\ i\%,\ 30)\\ 2.37 &= (F/P,\ i\%,\ 30)\end{aligned}$$

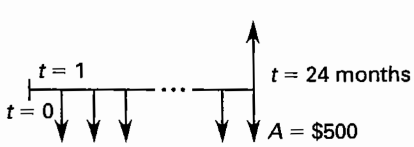
Use the formula for single payment compounding. If the table values were available, i could be determined by using linear interpolation. The effective rate per period is

$$\begin{aligned}2.37 &= (1+i)^{30}\\ i &= 0.02918 \quad (2.918\%)\end{aligned}$$

The nominal annual interest rate is twice this amount, or 5.8%.

Answer is D.

3. Create a cash flow diagram.

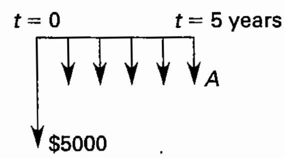


Use the uniform series compound amount discount factor.

$$\begin{aligned}F &= A(F/A,\ i\%,\ n)\\ A &= \$500\\ i &= \frac{12\%}{12\ \text{compounding periods per year}} = 1\%\\ n &= (2\ \text{years})(12\ \text{compounding periods per year})\\ &= 24\ \text{compounding periods}\\ F &= (\$500)(F/A,\ 1\%,\ 24) = (\$500)\left(\frac{(1+0.01)^{24}-1}{0.01}\right)\\ &= (\$500)(26.9735)\\ &= \$13{,}487 \quad (\$13{,}500)\end{aligned}$$

Answer is B.

4. Create a cash flow diagram.

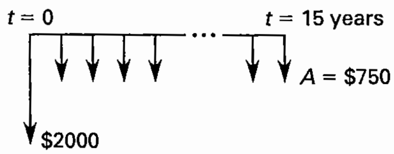


Use the capital recovery discount factor.

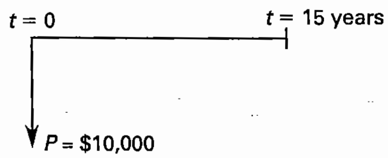
$$\begin{aligned}A &= P(A/P,\ i\%,\ n)\\ P &= \$25{,}000 - \$5000 = \$20{,}000\\ i &= \frac{6\%}{12\ \text{compounding periods per year}} = 0.5\%\\ n &= (5\ \text{years})(12\ \text{months per year}) = 60\\ A &= (\$20{,}000)(A/P,\ 0.5\%,\ 60)\\ &= (\$20{,}000)\left(\frac{(0.005)(1+0.005)^{60}}{(1+0.005)^{60}-1}\right)\\ &= (\$20{,}000)(0.0193)\\ &= \$386 \quad (\$400)\end{aligned}$$

Answer is B.

5. Create cash flow diagrams to compare the options.



(a) time payments



(b) single payment

Use the uniform series present worth discount factor.

$$P = \$10{,}000 - \$2000 = \$8000$$
$$A = \$750$$
$$n = 15$$
$$P = A(P/A,\ i\%,\ n)$$
$$\$8000 = (\$750)(P/A,\ i\%,\ 15)$$
$$10.67 = (P/A,\ i\%,\ 15)$$

From the factor tables, $i\%$ is below 6%. The formula for uniform series present worth can be used to determine the interest rate more closely.

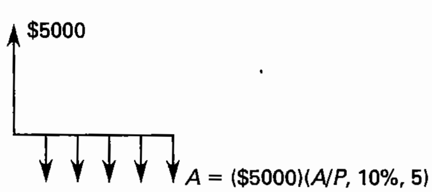
$$(P/A,\ i\%,\ 15) = \frac{(1+i)^n - 1}{i(1+i)^n}$$
$$10.67 = \frac{(1+i)^{15} - 1}{i(1+i)^{15}}$$

By trial and error,

$$i = 0.0461 \quad (4.61\%)$$

Answer is B.

6. Create a cash flow diagram.



Find the amount you will pay each year.

$$P = \$5000$$
$$i = 10\%$$
$$n = 5$$
$$(A/P,\ 10\%,\ 5) = 0.2638$$
$$A = P(A/P,\ i\%,\ n)$$
$$= (\$5000)(0.2638)$$
$$= \$1319$$

The interest paid at the end of the first year is

$$(\$5000)(0.10) = \$500$$

The principal left after the first year is

$$\$5000 - (\$1319 - \$500) = \$4181$$

The interest paid at the end of the second year is

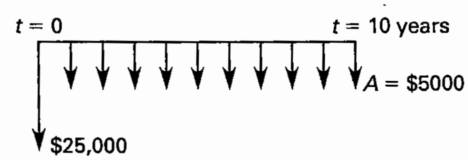
$$(\$4181)(0.10) = \$418$$

The total interest paid at the end of two years is

$$\$500 + \$418 = \$918$$

Answer is D.

7. Create a cash flow diagram.



Use the uniform series present worth factor.

$$A = \$5000$$
$$i = 10\%$$
$$n = 10 \text{ years}$$
$$(P/A,\ 10\%,\ 10) = 6.1446$$
$$P = A(P/A,\ i\%,\ n) + \$25{,}000$$
$$= (\$5000)(6.1446) + \$25{,}000$$
$$= \$55{,}723 \quad (\$55{,}700)$$

Answer is A.

Economics

8. The amount that you will pay your friend is

$$\begin{aligned} F &= P(F/P, i\%, n) \\ &= (\$3500)(F/P, 1.5\%, 12) \\ &= (\$3500)(1.015)^{12} \\ &= (\$3500)(1.1956) \\ &= \$4185 \end{aligned}$$

The amount that you would have paid the bank is

$$\begin{aligned} F &= P(F/P, i\%, n) \\ &= (\$3500)(F/P, 18\%, 1) \\ &= (\$3500)(1.18) \\ &= \$4130 \end{aligned}$$

The difference is

$$\$4185 - \$4130 = \$55$$

You have paid your friend \$55 more than you would have paid the bank.

Answer is A.

9. The future worth of \$25,000 10 years from now is

$$\begin{aligned} F &= P(F/P, 8\%, 10) = (\$25,000)(2.1589) \\ &= \$53,973 \quad (\$54,000) \end{aligned}$$

Answer is C.

10. The amount owed at the end of four years is

$$\begin{aligned} F &= P(F/P, 10\%, 4) \\ &= (\$10,000)(1.4641) \\ &= \$14,641 \end{aligned}$$

The amount owed after the \$3000 payment is

$$\$14,641 - \$3000 = \$11,641 \quad (\$11,700)$$

Answer is D.

11. Determine the future worth of \$20,000 25 years from now.

$$\begin{aligned} F &= P(F/P, 6\%, 25) \\ &= (\$20,000)(4.2919) \\ &= \$85,838 \quad (\$86,000) \end{aligned}$$

Answer is B.

12. To find the present worth of maintenance, use both the uniform gradient and the uniform series factors. Notice that both factors are evaluated for 10 years.

$$\begin{aligned} P &= A(P/A, 8\%, 10) + G(P/G, 8\%, 10) \\ &= (\$500)(6.7101) + (\$50)(25.9768) \\ &= \$4654 \quad (\$4700) \end{aligned}$$

Answer is B.

13. Use the uniform gradient uniform series equivalent factor to determine the effective annual cost.

$$\begin{aligned} A &= A_1 + G(A/G, 6\%, 10) \\ &= \$1000 + (\$500)(4.0220) \\ &= \$3011 \quad (\$3000) \end{aligned}$$

Answer is B.

14. Use the future worth continuous compounding factor.

$$\begin{aligned} F &= Pe^{rn} \\ &= \$10,000e^{(0.0525)(5)} \\ &= \$13,002 \quad (\$13,000) \end{aligned}$$

Answer is C.

15. Daily compounding is essentially equivalent to continuous compounding. Use the present worth equivalent factor for continuous compounding interest.

$$\begin{aligned} P &= Fe^{-rn} \\ &= \$5000e^{-(0.055)(5)} \\ &= \$3798 \quad (\$3800) \end{aligned}$$

Answer is B.

Economics